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SINGULAR POINTS OF DEFORMATION PROCESS AND CREEP BUCKLING OF A CYLINDRICAL SHELL

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To analyze buckling of rheological systems, a variant of the pseudo-bifurcational approach was presented in [1] which refines the earlier version [2] in its statement of the problem. The basis of the approach is the concept of a singular (or pseudo-bifurcational) point of the formation process. When this point is attained, the system reacts critically to specifying the increments of the higher derivatives of the deflection as the initial data for disturbed motion. The lower derivatives and the deflection itself increase unlimitedly here. The first point of the obtained sequence coincides with the criterion in [3].

The criterion [1] is used in the present study to solve three-dimensional problems. The basis of the solution is the elastic equivalent method, the essence of which is to split the problem into two problems. In the first problem the critical load of the corresponding elastic structure is calculated. In the second problem , which is connected with defining the relation only, a certain modulus expressing the increment of stresses in terms of increment of strains upon attainment of a singular point is found. Replacement of the Young's modulus in the first problem by the determined one (modulus of the elastic equivalent) gives the final solution.

To reveal the singular points, a system of equations is set up, which we will call the defining one. In the case of finding the singular points directly (which is possible in the simplest cases), such a system is formed for the increments of deflection and its derivatives with the help of equilibrium equations. In the elastic equivalent method the defining system connects creep strain increments with its derivatives with respect to time.

In contrast to [1] we will number the singular points starting from the first order. Thus, a pseudo-bifurcational point of order N (in the refined statement) corresponds to a singular point of order N + 1.

1. Let us write the relation of strain theory [4]

$$\varepsilon_{ij} = (\varepsilon/S)S_{ij}. \tag{1.1}$$

Here ε_{ij} is the strain and S_{ij} is the stress deviator:

$$S_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3; \tag{1.2}$$

S and ε are the intensities of the corresponding quantities

$$S^2 = S_{ij}S_{ij}; \quad \varepsilon^2 = \varepsilon_{ij}\varepsilon_{ij}. \tag{1.3}$$

We choose the defining relation in the power form

$$\dot{p}p^{\alpha} = AS^n, \tag{1.4}$$

$$p_{ij} = \varepsilon_{ij} - S_{ij}/(2G); \tag{1.5}$$

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$$p^2 = p_{ij}p_{ij}. \tag{1.6}$$

Convolution of (1.5) with (1.1) to (1.3), with (1.6) taken into account, yields $p = \varepsilon - S/(2G)$. With this equality it follows from (1.1) and (1.5) that

$$p_{ij} = (p/S)S_{ij}. \tag{1.7}$$

To derive the defining system of equations specifying singular points, certain identities will be required. We will confine ourselves to consideration of the basic process with constant stress deviators S_{ij} = const.

The first group of identities relates to the parameters of the basic process. Let us prove that

$$p_{ij}^{(k)} = p_{ij}(p^{(k)}/p),\ k = 1, 2, 3, \ldots \tag{1.8}$$

The index (k) indicates the order of the derivative with respect to time. Differentiating (1.7) k times with S_{ij}/S = const, we have

$$p_{ij}^{(k)} = (S_{ij}/S)p^{(k)},\ k = 1, 2, 3, \ldots$$

Replacing S_{ij}/S by p_{ij}/p in accordance with formula (1.7), we obtain the required identity (1.8) immediately.

One of the basic quantities in the derivation will be the tensor quantity

$$K_{ijmn} = S_{ij}S_{mn}/S^2 = p_{ij}p_{mn}/p^2, \tag{1.9}$$

having the property

$$K_{ijkl}K_{klmn} = K_{ijmn} \tag{1.10}$$

The second group of identities relates to the parameters of the disturbed process. Using (1.6) and (1.8), we can prove that

$$\Delta p^{(k)} = \Delta p_{mn}^{(k)}p_{mn}/p,\ (k = 0, 1, 2, \ldots). \tag{1.11}$$

Multiplying (1.11) by S_{ij}, we find

$$S_{ij}\Delta p^{(k)} = (p_{ij}p_{mn}/p^2)S\Delta p_{mn}^{(k)} = K_{ijmn}S\Delta p_{mn}^{(k)} \tag{1.12}$$

One more necessary identity follows from (1.7), (1.9), and (1.12)

$$\Delta p^{(k)}p_{ij} = pK_{ijmn}\Delta p_{mn}^{(k)} \tag{1.13}$$

To ascertain the regularity of formation of the system of equations and the elastic equivalent matrix chosen, it turns out that a few equations are sufficient for consideration. We confine ourselves to the third order of the system.

We differentiate the equality (1.7) twice with respect to time:

$$\dot{S}_{ij}p + S_{ij}\dot{p} = \dot{p}_{ij}S + p_{ij}\dot{S}; \tag{1.14}$$

$$\ddot{S}_{ij}p + 2\dot{S}_{ij}\dot{p} + S_{ij}\ddot{p} = \ddot{p}_{ij}S + 2\dot{p}_{ij}\dot{S} + p_{ij}\ddot{S}, \tag{1.15}$$

and also the defining relation (1.4):

$$\ddot{p}p^{\alpha} + \alpha p^{\alpha-1}\dot{p}^2 = An\dot{S}S^{n-1}; \tag{1.16}$$

$$\dddot{p}p^{\alpha} + 3\alpha p^{\alpha-1}\dot{p}\ddot{p} + \alpha(\alpha - 1)p^{\alpha-1}\dot{p}^3 = An(S^{n-1}\ddot{S} + (n - 1)\dot{S}^2). \tag{1.17}$$

We linearize (1.7), (1.14), and (1.15) taking account of the assumed restriction $\dot{S}_{ij} = 0$:

$$p\Delta S_{ij} + S_{ij}\Delta p = p_{ij}\Delta S + S\Delta p_{ij}; \tag{1.18}$$

$$p\Delta\dot{S}_{ij} + \dot{p}\Delta S_{ij} + S_{ij}\Delta\dot{p} = p_{ij}\Delta\dot{S} + \dot{p}_{ij}\Delta S + S\Delta\dot{p}_{ij}; \tag{1.19}$$

$$p\Delta\ddot{S}_{ij} + 2\dot{p}\Delta\dot{S}_{ij} + S_{ij}\Delta\ddot{p} + \ddot{p}\Delta S_{ij} = p_{ij}\Delta\ddot{S} + 2\dot{p}_{ij}\Delta\dot{S} + \ddot{p}_{ij}\Delta S + S\Delta\ddot{p}_{ij}. \tag{1.20}$$

Let us eliminate the increments ΔS, $\Delta\dot{S}$, and $\Delta\ddot{S}$ from the system (1.18) to (1.20). To this end we linearize (1.4), (1.16), and (1.17):

$$\Delta S = S(p\Delta\dot{p} + \alpha\dot{p}\Delta p)/(n\dot{p}p); \tag{1.21}$$

$$\Delta\dot{S} = S(p^2\Delta\ddot{p} + 2\alpha p\dot{p}\Delta\dot{p} - \alpha\dot{p}^2\Delta p)/(n\dot{p}p^2); \tag{1.22}$$

$$\Delta\ddot{S} = S(p^3\Delta\dddot{p} + 3\alpha p^2\dot{p}\Delta\ddot{p} - 3\alpha p\dot{p}^2\Delta\dot{p} + \alpha(\alpha + 1)\dot{p}^3\Delta p)/(n\dot{p}p^3). \tag{1.23}$$

The basic problem of the elastic equivalent method is that of defining the relation between increments of stressed and strain corresponding to a singular point. For convenience of calculations we define first the relationship between the increments of creep stresses and strains. We assume that the following relation is valid:

$$\Delta S_{ij}^{(k)} = (S/p)(aK_{ijmn} + b\delta_{im}\delta_{jn})\Delta p_{mn}^{(k)}, \tag{1.24}$$

where a and b are unknown coefficients.

Let us transform the right-hand sides of the system (1.18) to (1.20). With the help of identities (1.8) we replace $\dot{p}_{ij}$ by $(\dot{p}/p)p_{ij}$ and $\ddot{p}_{ij}$ by $(\ddot{p}/p)p_{ij}$. Then we eliminate the expressions $p_{ij}\Delta S^{(k)}$ (k = 0, 1, 2, ...) from the system by utilizing the equalities (1.21) to (1.23) transformed with the identities (1.13) taken into account.

We express the increments $\Delta S_{ij}^{(k)}$ on the left-hand side of the system (1.18) to (1.20) in terms of $\Delta p_{mn}^{(k)}$ with the help of (1.24). Utilizing identities (1.12), after certain transformations we obtain the system sought for:

$$\Delta p_{mn}[(\alpha/n - a - 1)K_{ijmn} + (1 - b)\delta_{im}\delta_{jn}] + (p/\dot{p})K_{ijmn}\Delta\dot{p}_{mn}/n = 0; \tag{1.25}$$

$$\Delta p_{mn}(\dot{p}/p)(aK_{ijmn} + b\delta_{im}\delta_{jn}) - \Delta\dot{p}_{mn}\{[(1 + 2\alpha)/n - 1 - a]K_{ijmn} + (1 - b)\delta_{im}\delta_{jn}\} - (p/\dot{p})K_{ijmn}\Delta\ddot{p}_{mn}/n = 0; \tag{1.26}$$

$$\Delta p_{mn}(\dot{p}/p)^2\alpha(aK_{ijmn} + b\delta_{im}\delta_{jn}) - 2\Delta\dot{p}_{mn}(\dot{p}/p)(aK_{ijmn} + b\delta_{im}\delta_{jn})$$
$$+ \Delta\ddot{p}_{mn}\{[(3\alpha + 2)/n - 1 - a]K_{ijmn} + (1 - b)\delta_{im}\delta_{jn}\} + (p/\dot{p})K_{ijmn}\Delta\dddot{p}_{mn}/n = 0. \quad (1.27)$$

If two or three more equations of the system are deduced by raising the order of the defining relation, one can determine the regularity of their formation. Thus, we write the N-th equation of the system (1.25) to (1.27)

$$(aK_{ijmn} + b\delta_{im}\delta_{jn})\sum_{k=0}^{N-2} C_k^{N-1}\psi_{N-2-k}\dot{p}^{N-1-k}\Delta p_{mn}^{(k)}/p - \Delta p_{mn}^{(N-1)}\{[(N$$
$$- 1 + N\alpha)/n - 1 - a]K_{ijmn} + (1 - b)\delta_{im}\delta_{jn}\} - (p/\dot{p})K_{ijmn}\Delta p_{mn}^{(N)}/n = 0,$$

where the functions ψ_k (k = 0, 1, 2, ..., N) have the form

$$\psi_0 = 1, \psi_1 = -\alpha/p, \psi_2 = \alpha(1 + 2\alpha)/p^2, \psi_3 = -\alpha(1 + 2\alpha)(2 + 3\alpha)/p^3...,$$
$$\psi_{N+1} = -\psi_N[(N + 1)\alpha + N]/p, N = 0,1,2,...$$

System (1.25) to (1.27) defines three singular points. We will find singular point of the N-th order from the condition of vanishing of the determinant of system of N tensor equations with unknown and known quantities $\Delta p_{ij}^{(k)}$ (k = 0, 1, 2, ..., N-1) and $\Delta p_{ij}^{(N)}$. To this end we will utilize the following method for setting the determinant. Putting $\Delta p_{ij}^{(N)} = 0$, we transform the defining system to the homogeneous one. We eliminate the unknowns $\Delta p_{ij}^{(k)}$ (k = 1, 2, ..., N-1) in consecutive order and obtain the equation for Δp_{ij}:

$$\Delta p_{ij}A_{ijmn} = 0. \quad (1.28)$$

Singular points are determinate from the condition of degeneration of the defining system, which is equivalent to $A_{ijmn} = 0$ in view of the independence of the components Δkp_{ij}. The unknown coefficients a and b of the elastic equivalent having the form (1.25) are determined from this equality.

Let us determine the singular point of the first order. We put $\Delta\dot{p}_{mn} = 0$. Equation (1.25) implies (1.28) immediately. The quantities K_{ijmn} and $\delta_{im}\delta_{jn}$ cannot be proportional. Equating the corresponding factors to zero separately, we obtain $a_1 = \alpha_1 = \alpha/n-1$ and $b_1 = 1$.

To determine the singular point of the second order, it is necessary to consider the system (1.25) and (1.26) with $\Delta\dot{p}_{mn} = 0$. Having eliminated Δp_{mn}, we reduce the system to the form (1.28). Having expressed $\Delta\dot{p}_{mn}K_{ijmn}$ from (1.26) and substituted it into (1.25), we have

$$\Delta p_{mn}\{K_{ijmn}[\alpha - z + 1 - b + z/(1 + 2\alpha - z)] + n\delta_{im}\delta_{jn}(1 - b)\} = 0,$$

where

$$z = (a + b)n. \quad (1.29)$$

Setting the coefficients of $\delta_{im}\delta_{jn}$ equal to zero, we get b = 1. Hence the coefficient of K_{ijmn} takes the form $[z^2 - 3\alpha z + \alpha(1 + 2\alpha)]/(1 + 2\alpha - z)$. The numerator of this expression coincides with the polynomial B_2 [1]; consequently, for z we have the solution $z = \xi_2$. Thus, for the singular point of the second order we determine the coefficients of the elastic equivalent matrix $a_2 = \xi_2/n - 1$ and $b_2 = 1$.

$$a_2 = \xi_2/n - 1, b_2 = 1.$$

Let us determine singular point of the third order. We assume $\Delta\dddot{p}_{mn} = 0$ in equation (1.27) and eliminate $\Delta\ddot{p}_{mn}K_{ijmn}$ and $\Delta\dot{p}_{mn}K_{ijmn}$ from (1.26) and (1.27). Omitting simple calculations, we have

$$\Delta p_{mn}\{K_{ijmn}[\alpha - z + n(1 - b) + z(4\alpha + 2 - z)/[2z$$

$$+ (1 + 2\alpha - z + n(1 - b))(2 + 3\alpha - z)]] + \delta_{im}\delta_{jn}n(1 - b)\} = 0.$$

Equating the coefficient of $\delta_{im}\delta_{jn}$ to zero, we get b = 1. Hence the factor of K_{ijmn} takes the form

$$[-z^3 + 6z^2 - \alpha(4 + 11\alpha)z + \alpha(2\alpha + 1)(3\alpha + 2)]/[2z + (1 + 2\alpha - z)(2 + 3\alpha - z)].$$

Numerator of this expression is the polynom B_3 [1] of variable z. Consequently for the critical point of the third order we have $z = \xi_3$. Taking into account the notation (1.29), we obtain $a_3 = \xi_3/n - 1$ and $b_3 = 1$.

In general case it is obvious that for the singular point of the N-th order the coefficients of the elastic equivalent matrix have the form

$$a_N = \xi_N/n - 1,\ b_N = 1. \tag{1.30}$$

The quantity ξ_N depends on α and is determined numerically as a root of the polynomial B_N. In the particular case $\xi_1 = \alpha$ and $\xi_2 \doteq (3\alpha \pm (\alpha^2 - 4\alpha)^{1/2})/2$.

Now we reduce relation (1.24) obtained in the elastic equivalent to the more natural form

$$\Delta S_{ij}{}^{(k)} = 2G_{ijmn}\Delta\varepsilon_{mn}{}^{(k)} \tag{1.31}$$

To this end, by utilizing the equality (1.5) written in increments and substituting the determined coefficients (1.30) we reveal $\Delta\varepsilon_{mn}$ from (1.24).

$$\Delta S^{(k)}_{mn}\{\delta_{im}\delta_{jn} + (S/p)(a_N K_{ijmn} + \delta_{im}\delta_{jn})/(2G)\} = (S/p)(a_N K_{ijmn} + \delta_{im}\delta_{jn})\Delta\varepsilon^{(k)}_{mn}. \tag{1.32}$$

Solving (1.32) for $S_{mn}{}^{(k)}$, we obtain the relation (1.31) with the matrix

$$G_{ijmn} = CK_{ijmn} + B\delta_{im}\delta_{jn}/2; \tag{1.33}$$

$$C = \frac{GpS(\xi_N - n)}{\varepsilon(2Gpn + S\xi_N)},\ B = \frac{S}{\varepsilon}. \tag{1.34}$$

The matrix (1.33) with coefficients (1.34) defines the elastic equivalent of media, which corresponds to the singular point of the N-th order.

2. We illustrate the application of the singular-point theory for the analysis of buckling phenomenon of thin-walled structures using the case of a cylindrical shell under axial compression. To this end we reduce the elastic equivalent matrix (1.33) corresponding to an arbitrary state of stresses to the form for the analysis of plane stresses, where the indices run from 1 to 2 only:

$$\Delta\sigma_{ij} = E_{ijmn}\Delta\varepsilon_{mn},\ i, j, m, n = 1, 2. \tag{2.1}$$

With $\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$ Eqs. (1.2) can be solved for the stresses:

$$\Delta\sigma_{ij} = S_{ij} + S_{kk}\sigma_{ij},\ i, j, k = 1, 2. \tag{2.2}$$

Simple calculations [2] with (2.2) and the condition of incompressibility $\Delta\varepsilon_{kk} = 0$ (k = 1, 2, 3) enable us to express the matrix E_{ijmn} in terms of the coefficients B and C or the matrix G_{ijmn}:

$$E_{ijmn} = 2C\sigma_{ij}\sigma_{mn}/S^2 + B(\delta_{im}\delta_{jn} + \delta_{ij}\delta_{mn}),\ i,j,m,n = 1,2. \tag{2.3}$$

Note that the intensity of the stress deviator S appearing in (2.3) is calculated from the previous formula (1.3) in which summation is carried out over all components of the tensor (i, j = 1, 2, 3).

We consider buckling phenomenon of a simply supported cylindrical shell of length l, wall thickness h, and radius R under axial compression. We assume that a relationship exists between the stresses and strains having the form (2.1) with the constant matrix (2.3). For simplicity the quantities B and C may be temporarily assumed to be constant, so that relation (2.1) will resemble outwardly Hooke's law for an anisotropic body (actually such an anisotropy does not exist in real elasticity). Let us determine the critical load in Euler's sense. We write the strain compatibility and equilibrium equations for the increments [2]:

$$\Delta\varepsilon_{1,22} + \Delta\varepsilon_{2,11} - 2\Delta\gamma_{,12} + W_{,11}/R = 0,\ \Delta M_{ij,ij} + N_{ij}W_{,ij} - \Delta N_{22}/R = 0. \tag{2.4}$$

Here ε_1, ε_2, and γ are the strains of the middle surface, W is the deflection of the shell (the sign Δ may be omitted for the prebuckling state of the shell for which the deflection equals zero), the comma denotes derivative with respect to the coordinate, and x_1 and x_2 are longitudinal and circumferential coordinates. The increments of the moments and forces are expressed in terms of stress increments by the formulas

$$\Delta M_{ij} = \int_{-h/2}^{h/2} \Delta\sigma_{ij} z\,dz,\ \Delta N_{ij} = \int_{-h/2}^{h/2} \Delta\sigma_{ij}\,dz. \tag{2.5}$$

We will utilize the geometrical relations

$$\Delta\varepsilon_{11} = \Delta\varepsilon_1 + zW_{,11},\ \Delta\varepsilon_{22} = \Delta\varepsilon_2 + zW_{,22},\ \Delta\varepsilon_{12} = \Delta\gamma + zW_{,12} \tag{2.6}$$

and the stress function

$$\Delta N_{11} = F_{,22},\ \Delta N_{22} = F_{,11},\ \Delta N_{12} = -F_{,12}. \tag{2.7}$$

For a cylindrical shell compressed by stresses σ_{11} along the generator we have $\sigma_{22} = 0$ and $S = \sqrt{2/3}\sigma_{11}$. In that case from the relation (2.1) with elastic equivalent matrix (2.3) it follows that

$$\Delta\sigma_{11} = (3C + 2B)\Delta\varepsilon_{11} + B\Delta\varepsilon_{22},\ \Delta\sigma_{22} = B(\Delta\varepsilon_{11} + 2\Delta\varepsilon_{22}),\ \Delta\sigma_{12} = B\Delta\varepsilon_{12}. \tag{2.8}$$

We rewrite system (2.4) expressing the increments of all quantities appearing there in terms of two functions: F and W. Let us transform the first equation of the system. We integrate (2.8) over the thickness bearing in mind the hypothesis of plane sections (2.6). We replace the stress integral by the forces by means of formulas (2.5) and solve the obtained system for the increments of strains of the middle surface:

$$\Delta\varepsilon_1 = \frac{2\Delta N_{11} - \Delta N_{22}}{3h(2C + B)},\ \Delta\varepsilon_2 = \frac{(3C + 2B)\Delta N_{22} - B\Delta N_{11}}{3Bh(2C + B)},\ \Delta\gamma = \frac{\Delta N_{12}}{Bh}. \tag{2.9}$$

We substitute expressions (2.9) into the first equation (2.4). Taking into account the replacement (2.7), we have

$$(3C + 2B)F_{,1111} + 4(3C + B)F_{,1122} + 2BF_{,2222} + 3B(2C + B)W_{,11}h/R = 0. \tag{2.10}$$

We transform the second equation of the system (2.4). Utilizing (2.6) and (2.8), we calculate the increments of the moments:

$$\Delta M_{11} = J((3C + 2B)W_{,11} + BW_{,22}),\ \Delta M_{22} = JB(W_{,11} + 2W_{,22}),$$
$$\Delta M_{12} = JBW_{,12}.$$

Here

$$J = \int_{-h/2}^{h/2} z^2 dz = h^3/12.$$

Taking into account that the force of axial compression for the basic state is $N_{11} = \sigma_{11}h$, we have the partial differential equation for deflection:

$$h^2[(3C + 2B)W_{,1111} + 4BW_{,1122} + 2BW_{,2222}]/12 + \sigma_{11}W_{,11} - F_{,11}/(Rh) = 0. \tag{2.11}$$

The critical load for axisymmetrical buckling can be easily determined from the system (2.10) and (2.11) if the variables W and F are assumed to be independent of circumferential coordinate x_2 and deflection is chosen in the form $w = U$ sing μx_1 ($\mu = m_1\pi/l$, where m_1 is the number of halfwaves along the generator). Omitting in this case simple calculations involving minimization of critical load with respect to μ, we find

$$\sigma_{11} = (h/R)\sqrt{B(2C + B)} = \sigma^*. \tag{2.12}$$

Comparing (2.8) with similar relations for an elastic incompressible material, we find that C = 0 and B = 2E/3 for elasticity. Hence (2.12) implies the well-known expression for the critical stress in an elastic shell $\sigma_0 = 2hE/(3R)$.

To analyze nonaxisymmetrical buckling we write the deflection W and the stress function F in the following form: $W = U \sin g\ \mu x_1 \sin \eta x_2$ and $F = \Phi \sin \mu x_1 \sin \eta = m_2/R$, where m_2 is a number of waves in circumferential direction). Hence the system (2.10) and (2.11) takes the form

$$(\varkappa + 12C\eta^2)\Phi - 3B(2C + B)hU/R = 0,\ \Phi/(Rh) + (h^2\varkappa/12 - \sigma_{11})U = 0, \tag{2.13}$$

where

$$\varkappa = \mu^2(3C + 2B) + 2\eta^2 B(2 + \eta^2/\mu^2). \tag{2.14}$$

Equating the determinant of the homogeneous system (2.13) to zero, we have

$$(h^2\varkappa/12 - \sigma_{11})(\varkappa + 12C\eta^2) + 3B(2C + B)/R^2 = 0.$$

Hence we find the expression for the stress:

$$\sigma_{11} = \frac{3B(2C + B)}{R^2(\varkappa + 12C\eta^2)} + \frac{h^2}{12}\varkappa. \tag{2.15}$$

Let us determine the minimum value of σ_{11} with respect to μ from the condition

$$\frac{\partial\sigma_{11}}{\partial\mu} = \frac{\partial\sigma_{11}}{\partial\varkappa}\frac{\partial\varkappa}{\partial\mu} = 0. \tag{2.16}$$

Two cases are possible here. Let the first factor $\partial\sigma_{11}/\partial\kappa$ be equal to zero. Differentiating (2.15), we find that the minimum value of σ_{11} occurs for $\kappa = 6\sigma^*/h^2 - 12C\eta^2$:

$$\sigma_{11} = \sigma^* - Ch^2\eta^2. \tag{2.17}$$

Consider another solution resulting form (2.16): $\partial\kappa/\partial\mu = 0$. Differentiating (2.14), we find that the extreme value occurs for the value $\mu^2 = \eta^2\sqrt{2B/(3C + 2B)}$ or $\kappa = 2\eta^2(\sqrt{2B(3C + 2B)} + 2B) = 2\eta^2 Z$ and corresponds to the quantity

$$\sigma_{11} = \frac{3B(B + 2C)}{2R^2(Z + 6C)\eta^2} + \frac{1}{6}h^2\eta^2 Z, \text{ where } Z = \sqrt{2B(3C + 2B)} + 2B.$$

Minimizing this expression with respect to the variable η, we find that the stress σ_{11} has the minimum value when $\eta^2 = 3/(Rh)\sqrt{B(B + 2C)/[Z(Z + 6C)]}$:

$$\sigma_{11} = \sigma^*\sqrt{Z/(Z + 6C)}. \tag{2.18}$$

If $C < 0$, then the load obtained for monaxisymmetrical buckling (2.17) or (2.18) is greater than that for axisymmetrical buckling $\sigma_{11} = \sigma^*$. This means that the nonaxisymmetrical case it impossible when $C < 0$.

When $C > 0$ it is necessary to consider solutions (2.17) and (2.18), choosing that one which corresponds to the lower load. The qualitative difference between these solutions consists in the buckling mode predicted. Since the function σ_{11} increases with increasing η according to (2.17), it is obviously necessary to take the lowest value $\eta = 1/R$ which corresponds to one wave or the lateral buckling of a shell like a rod. It is evident that when the ratio h/R is sufficiently large (within the limits assumed in the technical theory of shells) the solution (2.17) will be less than (2.18). This is to be expected: thick-walled long shells can buckle in a lateral way as rods do, but in the case of thin-walled shells, dents appear. However, practical calculations show that solution (2.18) is as a rule, realized.

3. Let us consider the experiment [5] on buckling of shells made of alloy D16T and subjected to axial compressive loads at 250°C (data for shells Nos. 18 to 32). The parameters α and n were not presented in [5]. Using the creep curves presented there, we determine the rheological constants α and n appearing in relation (1.4) using the approach of [4]. Replotting these curves in logarithmic coordinates and taking into account inevitable errors, we find the ranges of values of the unknown parameters: $0.6 < \alpha < 0.8$ and $3.5 < n < 4.2$. We set $\alpha = 0.75$ and $n = 4$.

We compare the theoretical and experimental results in the ω and ε_0 axes, where $\omega = \sigma_{11}/\sigma_0$ and $\varepsilon_0 = Ep_{11}/\sigma_0$. We rewrite (1.34) in terms of ε_0 and ω with (1.30) taken into account:

$$C = \frac{E\varepsilon_0\omega a_N}{3(\varepsilon_0 + \omega)(\varepsilon_0 + (a_N + 1)\omega)}, \quad B = \frac{2E\omega}{3(\varepsilon_0 + \omega)}. \tag{3.1}$$

The roots of the first few polynomials B_N, on which the singular points depend, have the following values [1].

$$\xi_1 = 0.75, \ \xi_3 = 1.865, \ \xi_5 = 2.998, \ \xi_7 = 4.140, \ \xi_9 = 5.286, \ \xi_{11} = 6.436. \tag{3.2}$$

The even polynomials have no roots [1]. For chosen rheological constants with (3.2) we calculate the values of the coefficients a_N (1.30) corresponding to the first singular points: $a_1 = -0.813$, $a_3 = -0.534$, $a_5 = -0.251$, $a_7 = 0.035$, $a_9 = 0.322$, and $a_{11} = 0.609$.

The buckling mode depends on sign of C. It can be easily seen from (3.1) that in this case for singular points of the 1st, 3rd, and 5th orders we have $C < 0$ and therefore axisymmetrical buckling occurs with the critical load (2.12). Substituting (3.1) into (2.12), we find

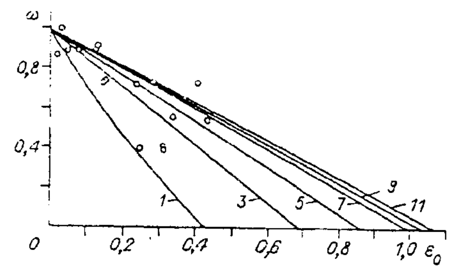

Fig. 1

$$\varepsilon_0^2 + \varepsilon_0\omega(a_N + 2) + (\omega^2 - 1)(a_N + 1) = 0.$$

The solution of the equation has the form

$$\varepsilon_0 = (\pm\sqrt{4(a_N + 1) + \omega^2 a_N^2} - \omega(a_N + 2))/2. \quad (3.3)$$

For instantaneous buckling ($\varepsilon_0 = 0$) corresponding to Euler load $\omega = 1$, from the two solutions we take the physically valid solution with plus sign. Curves 1, 3, and 5 (see Fig. 1) are plotted for singular points of the corresponding orders in accordance with solution (3.3).

The solution points ξ_7, ξ_9, and ξ_{11} correspond to nonaxisymmetrical buckling. Of the two formulas (2.17) and (2.18), which are valid for the critical parameters of the stress-strained state, as is found from calculations, formula (2.18) gives lower stress at the same creep strain. Curves for the singular points ξ_7, ξ_9 and ξ_{11} are plotted on the basis of the numerical solution of the equation following from (2.18), where the expressions for B and C (3.1) were substituted. The coordinates of the experimental points were calculated on the basis of the Euler load obtained from the experiment ($\sigma_0 = 98$ MPa). In this way the initial imperfections of the shells were taken into account approximately.

A characteristic thickening of the curves is seen, which corresponds to singular points of higher orders. Though there is no upper bound of the singular points, it should be noted that the thickening of the critical curves coincides with the experimental data (or is close to them in spite of their considerable scatter).

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